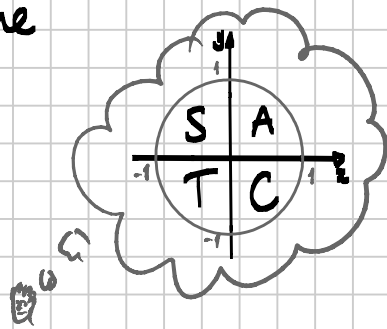


C2 Exercise 8C (CAST diagram and trig relations)

Note Title

28/04/2013

This is the CAST diagram:



It tells you which trig. function(s) are positive in each quadrant:

C $\cos \theta$ only

A all of them

S $\sin \theta$ only

T $\tan \theta$ only.

Notice the mnemonic CAST must begin in the 4th quadrant.

The textbook starts (perhaps more logically) in the first quadrant and uses the mnemonic ASTC.

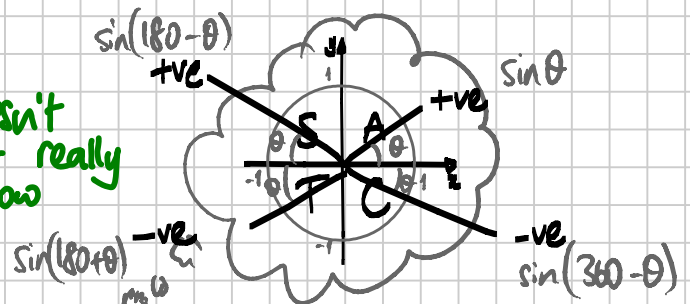
The book encourages you to learn nine rules about trig. functions:

$$\sin(180 - \theta) \equiv \sin \theta$$

$$\sin(180 + \theta) \equiv -\sin \theta$$

$$\sin(360 - \theta) \equiv -\sin \theta$$

but doesn't make it really clear how these come from the CAST diagram:

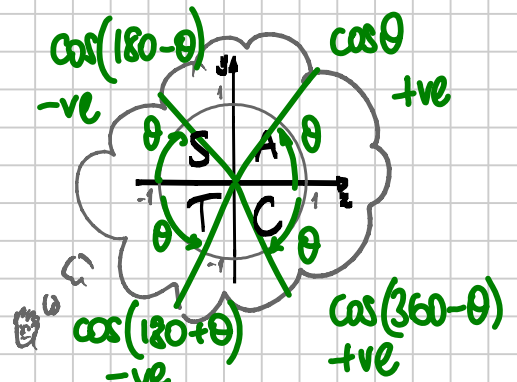


$$\cos(180 - \theta) = -\cos \theta$$

$$\cos(180 + \theta) = -\cos \theta$$

$$\cos(360 - \theta) = \cos \theta$$

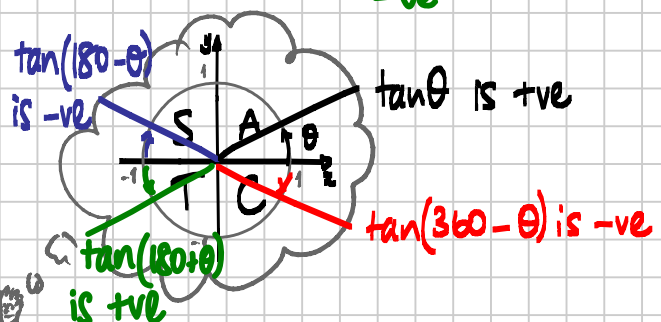
again they're all to do with $\cos \theta$ but are they + or -?



$$\tan(180 - \theta) = -\tan \theta$$

$$\tan(180 + \theta) = +\tan \theta$$

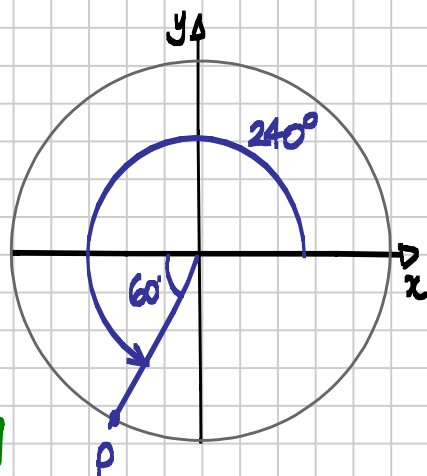
$$\tan(360 - \theta) = -\tan \theta$$



1 By drawing a sketch diagram express these in terms of the trig. function of a corresponding acute angle.

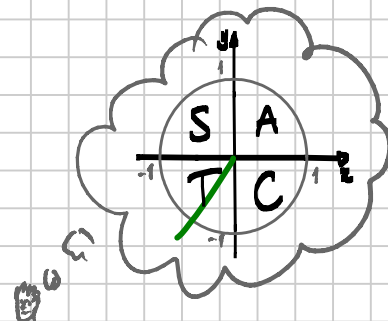
a $\sin 240^\circ$

step ①:
as you did
in exercise
8A find the
corresponding
acute angle



240° goes with 60°

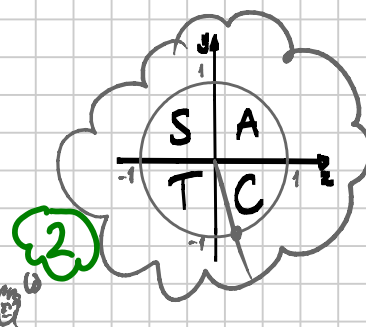
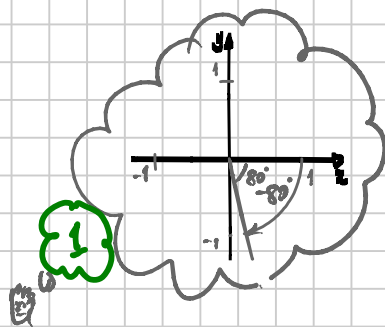
step ②: think 'CAST':



... is sine positive or negative at 240° ? negative

③ put them together $\Rightarrow \sin 240 = -\sin 60^\circ$

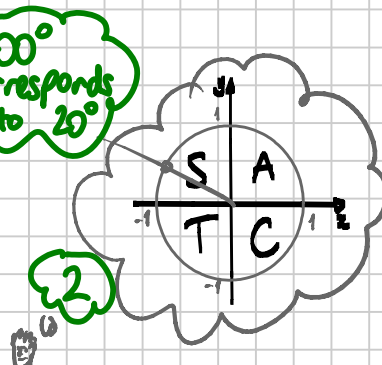
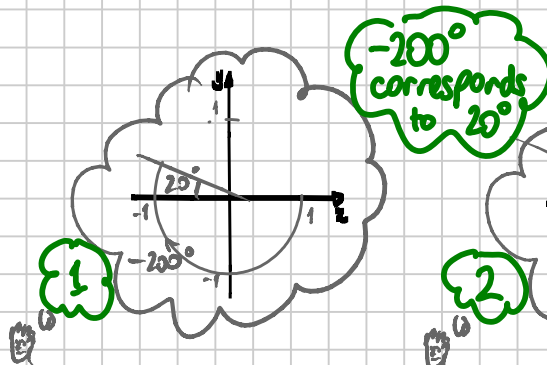
b $\sin(-80^\circ)$



sin -ve in here

$\sin(-80^\circ) = -\sin 80^\circ$

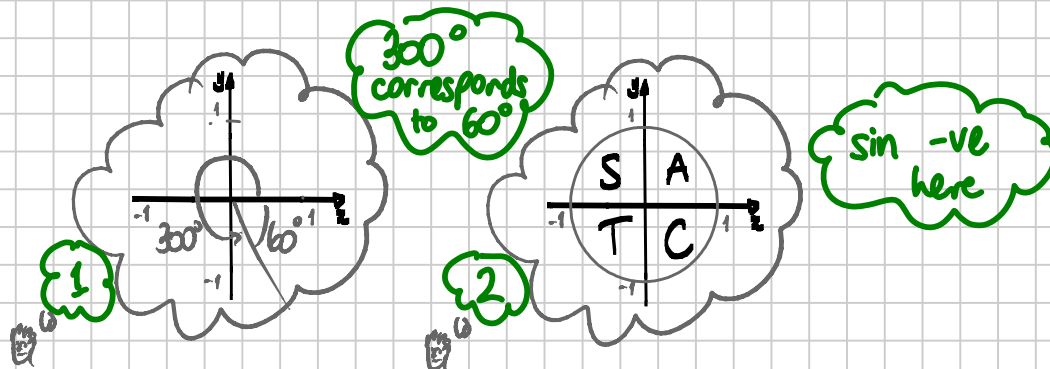
c $\sin(-200^\circ)$



sin +ve here

$\sin(-200^\circ) = \sin 20^\circ$

d $\sin(300)^\circ$

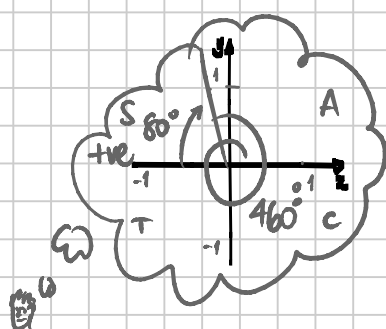


$$\sin(300)^\circ = -\sin 60^\circ$$

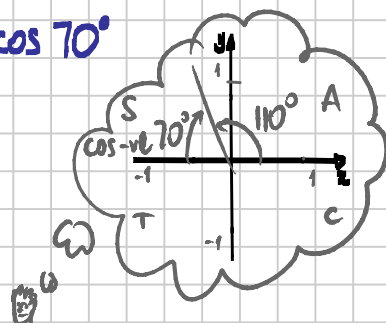
You can save a bit of time thinking of both in the same diagram:

e $\sin 460^\circ$

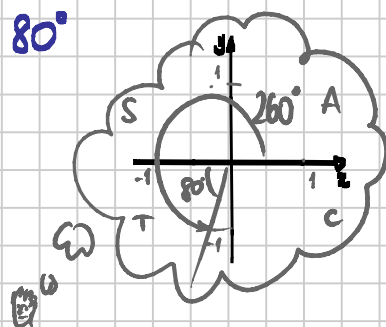
$$= \sin 80^\circ$$



f $\cos 110^\circ = -\cos 70^\circ$

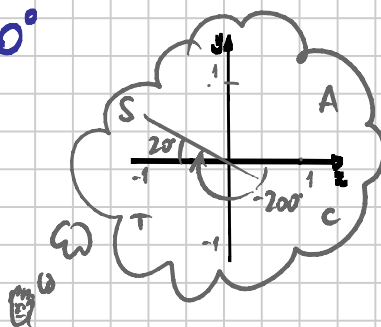


g $\cos 260^\circ = -\cos 80^\circ$

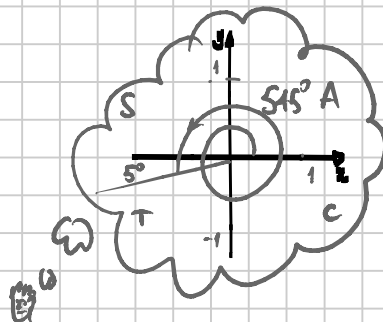


h $\cos -50^\circ = \cos 50^\circ$

i $\cos(-200)^\circ = -\cos 20^\circ$

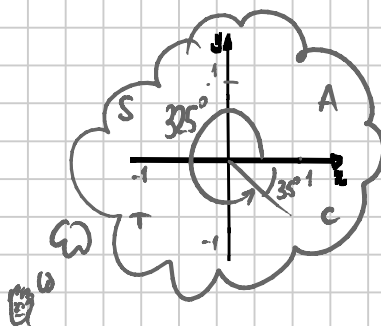


j $\cos(545)^\circ = -\cos 5^\circ$



k $\tan 100^\circ = -\tan 80^\circ$

l $\tan 325^\circ = -\tan 35^\circ$

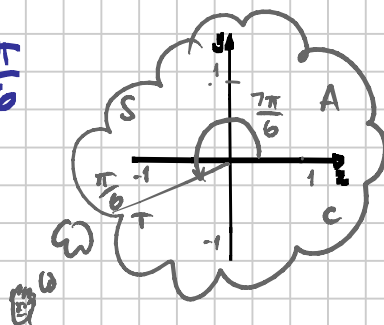


m $\tan(-30)^\circ = -\tan 30^\circ$

n $\tan(-175)^\circ = \tan 5^\circ$

o $\tan 600^\circ = \tan 240 = \tan 60^\circ$

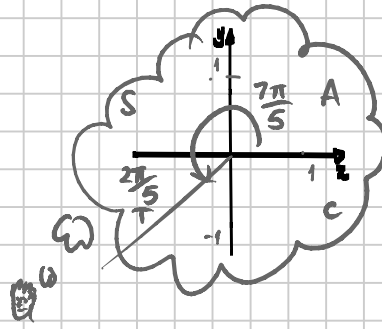
p $\sin \frac{7\pi}{6} = -\sin \frac{\pi}{6}$



q $\cos \frac{4\pi}{3} = -\cos \frac{\pi}{3}$

r $\cos\left(-\frac{3\pi}{4}\right) = -\cos \frac{\pi}{4}$

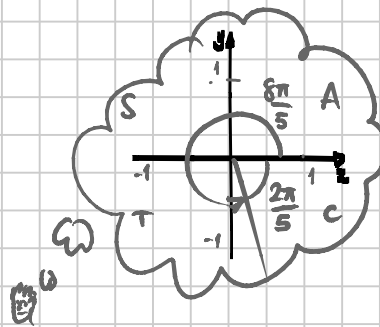
$$1s \quad \tan \frac{7\pi}{5} = \tan \frac{2\pi}{5}$$



$$t \quad \tan \left(-\frac{\pi}{3} \right) = -\tan \frac{\pi}{3}$$

$$u \quad \sin \left(\frac{15\pi}{16} \right) = \sin \frac{\pi}{16}$$

$$v \quad \cos \left(\frac{8\pi}{5} \right) = \cos \left(\frac{2\pi}{5} \right)$$

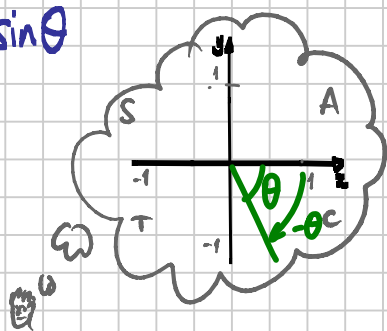


$$w \quad \sin \left(-\frac{6\pi}{7} \right) = -\sin \frac{\pi}{7}$$

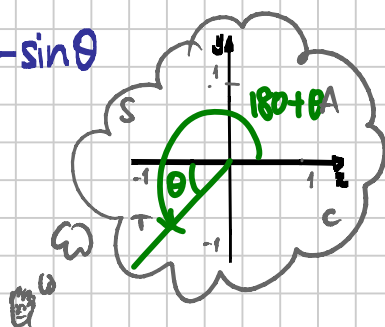
$$x \quad \tan \frac{15\pi}{8} = -\tan \frac{\pi}{8}$$

2 Given that θ is an acute angle measured in degrees express in terms of $\sin \theta$.

a $\sin(-\theta) = -\sin \theta$

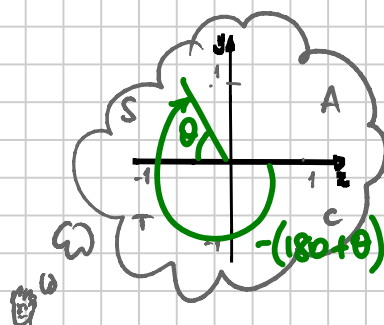


b $\sin(180 + \theta) = -\sin \theta$

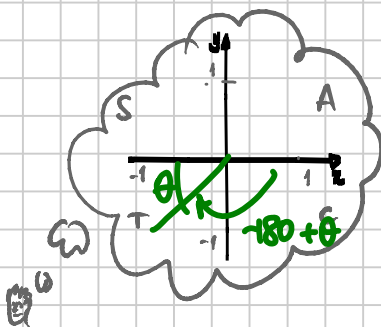


c $\sin(360 - \theta) = -\sin \theta$

d $\sin -(180 + \theta) = \sin \theta$

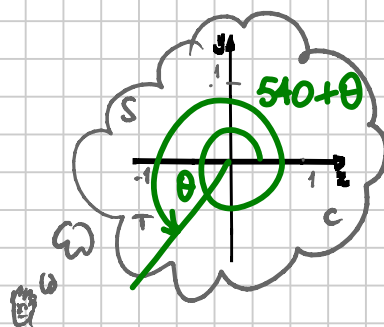


e $\sin(-180 + \theta) = -\sin \theta$

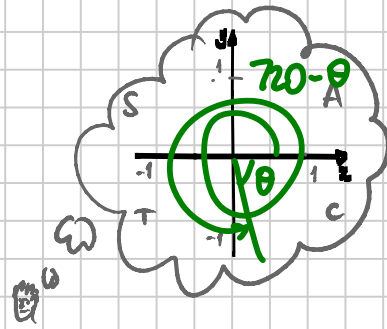


f $\sin(-360 + \theta) = \sin \theta$

g $\sin(540 + \theta) = -\sin \theta$



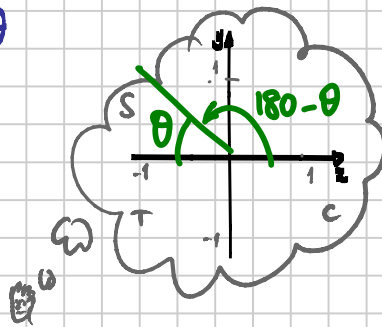
$$2h \quad \sin(720 - \theta) = -\sin \theta$$



$$i \quad \sin(\theta + 720) = \sin \theta$$

3 Given that θ is an acute angle measured in degrees express in terms of $\cos \theta$ or $\tan \theta$

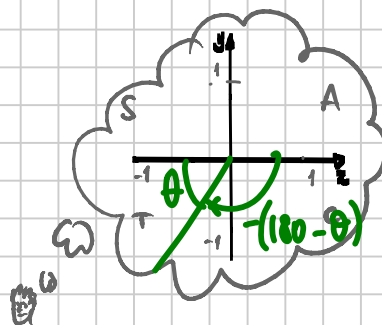
$$a \quad \cos(180 - \theta) = -\cos \theta$$



$$b \quad \cos(180 + \theta) = -\cos \theta$$

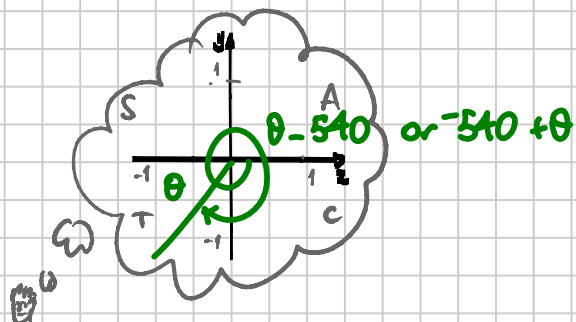
$$c \quad \cos(-\theta) = \cos \theta$$

$$d \quad \cos - (180 - \theta)^\circ = -\cos \theta$$

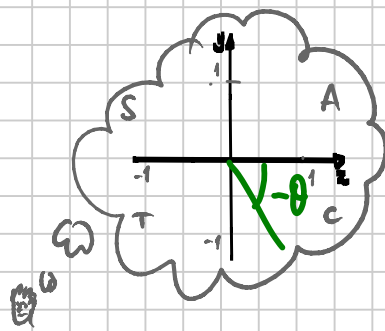


$$e \quad \cos(\theta - 360) = \cos \theta$$

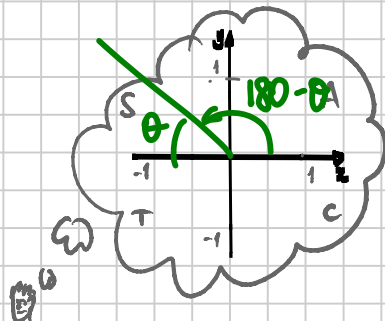
$$f \quad \cos(\theta - 540) = -\cos \theta$$



$$3g \quad \tan(-\theta) = -\tan \theta$$



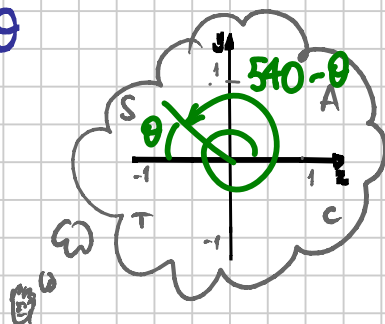
$$h \quad \tan(180 - \theta) = -\tan \theta$$



$$i \quad \tan(180 + \theta) = \tan \theta$$

$$j \quad \tan(-180 + \theta) = \tan \theta$$

$$k \quad \tan(540 - \theta) = -\tan \theta$$



$$l \quad \tan(\theta - 360) = \tan \theta$$

4 A function f is an 'even function' if $f(-\theta) = f(\theta)$

A function f is an 'odd function' if $f(-\theta) = -f(\theta)$

Use the answers above to determine whether \sin , \cos and \tan are even, odd or neither.

$$2a) \quad \sin(-\theta) = -\sin \theta$$

\Rightarrow sine is an odd function

$$3c) \quad \cos(-\theta) = \cos \theta$$

\Rightarrow cosine is an even function

$$3g) \quad \tan(-\theta) = -\tan \theta$$

\Rightarrow tangent is an odd function